

Towards the molecular workshop: entropy-driven designer molecules, entropy activation, and nanomechanical devices

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We propose a new class of designer molecules with functional units which are driven by *entropic* forces. This idea profits from the mechanically interlocked nature of topological molecules such as catenanes and rotaxanes, which allows for *mobile* elements whose accessible configuration space gives rise to entropic intramolecular forces. Such entropy-driven designer molecules open the possibility for externally controllable functional molecules and nanomechanical devices.

Complex molecules can be endowed with the distinct feature that they contain subunits which are linked to each other mechanically rather than chemically [1]. The investigation of the structure and properties of such interlocked *topological molecules* is subject of the growing field of chemical topology [2]; while speculations about the possibility of catenanes [27] (Olympic rings) date back to the early 20th century lectures of Willstätter, the actual synthesis of catenanes and rotaxanes [28] succeeded in 1958 [1]. Modern organic chemistry has seen the development of refined synthesis methods to generate topological molecules.

In parallel to the miniaturisation in electronics [3] and the possibility of manipulating single (bio)molecules [4], supramolecular chemistry which makes use of chemical topology properties is coming of age [5, 6]. Thus, rotaxane-type molecules are believed to be the building blocks for certain nanoscale machines and motors [7], so-called hermaphrodite molecules have been shown to perform linear relative motion (“contraction and stretching”) [8], and pirouetting molecules have been synthesised [9]. (In part, the inspiration for these new kinds of *designer molecules* comes from the biological machinery contained in cellular systems [10].) Moreover, topological molecules are thought to become components for molecular electronics switching devices in memory and computing applications [11, 12]. These molecular machines are usually of lower molecular weight, and their behaviour is essentially energy-dominated in the sense that their conformations and dynamical properties are governed by external and thermal activation in an energy landscape. The understanding of the physical properties and the theoretical modelling of such designer molecules and their natural biological counterparts has increasingly gained momentum, and the stage is already set for the next generation of applications [3-9, 11-18].

In this work, we introduce some basic concepts for functional molecules whose driving force is entropic rather than energetic. These molecules will be of higher molecular weight (hundred monomers or above) in order to provide sufficient degrees of freedom such that entropic effects can determine the behaviour of the molecule. The

potential for such *entropy-driven functional molecules* can be anticipated from the classical Gibbs Free energy

$$F = U - TS; \quad (1)$$

in functional molecules, F is minimised mainly by variation of the internal energy U representing the shape of the energy landscape of the functional unit. Here, we propose new types of molecules for which F is minimised by variations of the entropy S , while the energies and chemical bondings are left unchanged. The entropy-functional units of such molecules can be specifically controlled by external parameters like temperature, light flashes, or other electromagnetic fields [5, 6]. We note that DNA is already being studied as a macromolecular prototype building block for molecular machines [15].

To be more specific, and to gain some intuition about entropic effects in molecules, in Fig. 1 we depict a simple entropy-driven functional molecule, which consists of a linear, flexible polymer chain. The chain is constrained by a cyclic molecule which acts as a *slip-link*, i.e., it forces two given chain monomers close together such that two parts of the chain can freely slide through it [19, 20, 21]. Both ends of the chain are capped with large groups which act as *stoppers* to capture the slip-link. In addition, the loop threads another cyclic molecule, a *sliding ring* [20, 21], which represents its functional unit. In terms of the arc length of the loop, the sliding ring is subject to a periodic potential, the periodicity and shape of which are determined by the spatial extension and the specific chemical composition, respectively, of the chain monomers.

Suppose that, initially, the system is coupled to a heat bath with temperature T_0 at which the sliding ring is trapped in one of the valleys of this periodic potential along the loop, such that it is immovably placed on it. (The sliding ring merely acts to prevent the loop from escaping through the slip-link by configurational fluctuations of the polymer chain.) In this case, the free energy (1) of the system is minimised by maximising its entropy S ; we have shown in Ref. [20, 21] that, accordingly, the probability density function to find the loop

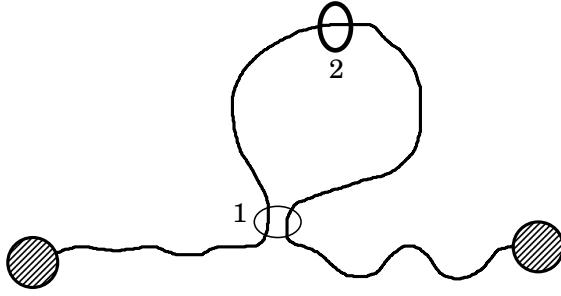


Figure 1: Linear polymer chain with stoppers at both ends. A permanent loop is formed by the slip-link, 1, holding two parts of the chain close together. On the loop, a sliding ring, 2, is placed.

with arc length ℓ is given by

$$p_0(\ell) = N\ell^{-3\nu+\sigma_4} \propto \ell^{-2.24}, \quad (2)$$

where N is a normalisation factor, $\nu \approx 0.588$ is the Flory exponent, and $\sigma_4 \approx -0.48$ is a topological exponent which characterises a vertex at which four segments of a flexible, self-avoiding chain are linked together [22]. The average loop size is given by

$$\langle \ell \rangle_0 = \int_a^L d\ell \ell p_0(\ell) \sim a, \quad (3)$$

where L is the arc length of the polymer chain and the length a represents the smallest possible size of the loop as given by the actual design (a is of the order of the monomer size). Thus, the loop is entropically favoured to be *tight*, and the whole compound closely behaves like a linear chain of length L .

A completely different picture results if the sliding ring is activated, e.g., by increasing the temperature T sufficiently above T_0 or by the influence of external electromagnetic fields, such that it can overcome the potential barriers of the periodic potential along the loop, and thus can freely slide on it. Since for each specific configuration of the polymer chain with loop length ℓ the sliding ring can now occupy ℓ different positions along the loop, the probability density function is modified to

$$p_1(\ell) = \ell p_0(\ell) \propto \ell^{-1.24}, \quad (4)$$

which implies

$$\langle \ell \rangle_1 = \int_a^L d\ell \ell p_1(\ell) \sim a^{0.24} L^{0.76}, \quad (5)$$

i.e., the additional degrees of freedom of the activated sliding ring result in the fact that the loop is not tight but grows with L . This entropic effect can be further enhanced by placing not only one, but two or more sliding rings on the loop, since $p_n(\ell) = \ell^n p_0(\ell)$ and $\langle \ell \rangle_n \sim L$ for $n \geq 2$ activated sliding rings. For several sliding rings, the loop will in fact be the dominating feature, i.e., the

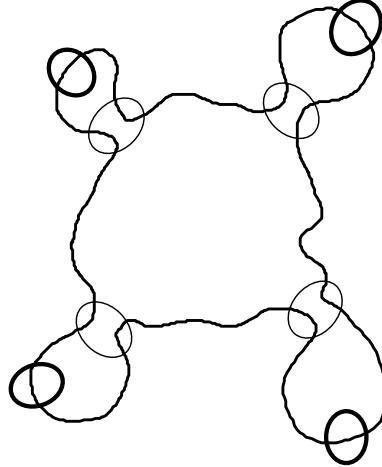


Figure 2: Central ring polymer from which a number of slip-links separate off fringe loops. Within the latter, sliding rings are placed. If the sliding rings are immobile, the central loop is the overall governing structure. If the sliding rings are activated and become mobile, the fringe loops tend to be large, which leads to a decrease of the mean extension of the compound.

compound behaves more and more like a ring polymer of length L as sliding rings are added. Thus, for a melt or solution of such molecules, the externally controllable activation of the sliding ring(s) leads to a topological, entropy-driven transition from linear chain to ring polymer.

The above statements are valid for long chains. We have obtained that chains with one loop can be regarded long for about 100 or more statistically independent units. If the chain is shorter, finite size effects come into play and diminish the degrees of freedom of the sliding rings, and therefore the influence of entropy. However, this might even be desirable if entropy-functionality is expected to change the properties of a system only slightly. In the above example, a shorter chain might exhibit only slightly increased entropy-swelling of the loop.

Similar entropic effects govern the configuration depicted in Fig. 2, in which a number of slip-links are placed along a ring polymer. Within each of the fringe loops, additional sliding rings are placed. If, initially, the sliding rings are immobile, the central loop consumes almost the entire length of the polymer ring. Conversely, if the sliding rings are activated, the fringe loops are entropically favoured to be large, which should decrease the mean extension of the entire compound drastically. It would be interesting to create melts or gels of such systems; as the single compound (for immobile sliding-rings) closely behaves like a ring polymer, the concatenation of a number of such compounds should require no new chemistry.

To see how energetic and entropic contributions compete with each other, consider the idealised rotaxane depicted in Fig. 3. Typically, a rotaxane consists of a ring, 1, on a backbone strand and is subject to a potential landscape $U_1(x)$ with a stable minimum at a position x_0

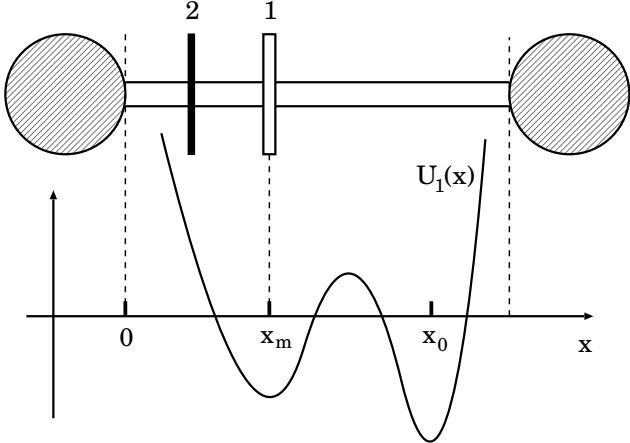


Figure 3: Modified rotaxane molecule. The usual rotaxane ring, 1, and the sliding ring, 2, are placed along the molecule strand, between the stopper balls (e.g., C₆₀ [6]). Without activated sliding ring, the rotaxane ring is subject to a potential landscape $U_1(x)$ (here sketched schematically with a stable position x_0 and a metastable position x_m). If the sliding ring is activated, it exerts an additional entropic force on the rotaxane ring, which tends to shift it to the right.

and a metastable minimum at another position x_m . A laser pulse can be used to move the rotaxane ring from x_0 to x_m before it relaxes back to x_0 . Since the potential barrier(s) between x_m and x_0 can be high, the refractory times can be fairly long (e.g., of the order of days) [5, 6, 7, 16]. In a modified rotaxane molecule, one could put an additional sliding ring which, if activated, can freely slide along the backbone of the rotaxane between the stopper and the rotaxane ring, like a particle of a one dimensional ideal gas. The free energy (1) of the system for a fixed position x of the rotaxane ring becomes

$$F(x) = U_1(x) - k_B T \log(x/a), \quad (6)$$

where k_B is Boltzmann's constant and x/a is the number of sites the sliding ring, 2, can take on. On the rotaxane ring therefore acts the effective force

$$K(x) = -F'(x) = -U'_1(x) + k_B T/x, \quad (7)$$

where the entropic contribution $k_B T/x$ tends to shift the rotaxane ring to the right. Thus, entropy-functional units could be used, e.g., to accelerate the often extremely long refractory times in rotaxanes. Conversely, if placed on the opposite side of the metastable rotaxane ring position and activated, the sliding ring could increase the refractory time and thus lead to a stabilisation of the metastable position of the rotaxane ring for the purpose of information storage. We note that it should be possible to synthesise long rod-like axles for entropy-functional rotaxane molecules by using translationally invariant homopolymer helices [23].

Finally, consider the nanomechanical device depicted in Fig. 4. According to the arrangement of the sliding rings 1 and 2, this compound exhibits the so far unique

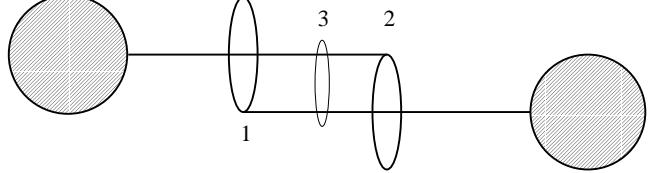


Figure 4: “Molecular muscle” consisting of two interlocked rings 1 and 2 with attached rod-like molecules. Within this structure, sliding rings, 3, can be placed, which, if activated, tend to contract the muscle by entropic forces.

feature of a molecule that it can slide *laterally*. Suggested as precursors of molecular muscles [8], this compound could be propelled with internal entropy-motors, which entropically adjust the elongation of the muscle. In the configuration shown in Fig. 4, the sliding ring 3 creates, if activated, an entropic force which tends to contract the “muscle”; at $T = 300$ K and on a typical scale $x = 10$ nm, the entropic force $k_B T/x$ is of the order of pN, and thus comparable to the force created in biological muscle cells [24]. Molecular muscles of such a make can be viewed as the nano-counterpart of macroscopic muscle models proposed by de Gennes [25], in which the contraction is based on the entropy difference between the isotropic and nematic phases in liquid crystalline elastomer films [26].

Entropy has so far been disregarded in the design of functional molecules, although the basis for this, i.e., the mechanically interlocked state of topological molecules, has been achieved for some time, and prototype molecules with sliding rings have been synthesised [6]. Entropy-functional units can be specifically controlled by external parameters, e.g., temperature and electromagnetic fields. Functional behaviour such as controlled transition from linear chain to ring polymer, swelling/de-swelling, switching in rotaxane-like compounds, and molecular motion (muscle contraction) could thus be achieved without changing the chemical structure of the involved compounds (the latter is done in designed biomolecular motors [13, 14, 15]). As another possible application, one might speculate whether the DNA helix-coil transition [23] in multiplication setups could be facilitated in the presence of pre-ring molecules which *in vitro* attach to an opened loop of the double strand and close, creating an entropy pressure which tends to open up the vicinal parts of the DNA which are still in the helix state. Finally, considering molecular motors, it would be interesting to design an externally controllable, purely entropy-driven rotating nanomotor.

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- [27] *catena* (lat.), the chain.
- [28] *rota* (lat.), the wheel; *axis* (lat.), the axle.